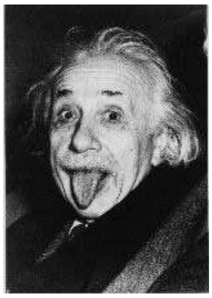
Formelsammlung Klassische Feldtheorie



8 Klassische Feldtheorie (im engeren Sinn)

	Klassische Mechanik	Klassische Feldtheorie
unabhängige Variable	Zeit t oder Eigenzeit τ	Raum(-Zeit) Koordinaten x^α $\alpha = 0, \ldots, d-1$ oder $\alpha = 1, \ldots d$
abhängige Variable	Koordinaten $q^i(t)$ oder $x^\alpha(\tau)$	Felder $\Phi^A(x^\alpha)$ Φ^A: Skalar-, Vektor-, Tensorfelder etc.
Ableitungen	$(d/dt)q^i(t) = \dot{q}^i(t)$ oder $\dot{x}^\alpha(\tau)$	partielle Ableitungen $\partial_\alpha \Phi^A(x)$
Lagrange-Funktion	$L = L(q^i, \dot{q}^i; t)$	$L = L(\Phi^A, \partial_\alpha \Phi^A; x^\alpha)$
Wirkung	$S[q] = \int dt\, L$	$S[\Phi] = \int d^d x\, L$
Variation	$q^i(t) \to q^i(t) + \delta q^i(t)$	$\Phi^A(x) \to \Phi^A(x) + \delta\Phi^A(x)$
Euler-Lagrange Gleichungen	$(\partial L/\partial q^i) - (d/dt)(\partial L/\partial \dot{q}^i) = 0$ $\frac{d}{dt} = \frac{\partial}{\partial t} + \dot{q}^i(t)\frac{\partial}{\partial q^i(t)} + \ldots$	$(\partial L/\partial \Phi^A) - (d/dx^\alpha)(\partial L/\partial(\partial_\alpha \Phi^A)) = 0$ $\frac{d}{dx^\alpha} := \frac{\partial}{\partial x^\alpha} + \partial_\alpha \Phi^A(x)\frac{\partial}{\partial \Phi(x)} + \ldots$

1. Reelles Skalarfeld Φ im Minkowski-Raum

   (a) Freies masseloses Feld

$$\begin{aligned} L(\Phi, \partial_\alpha \Phi) &= -\tfrac{1}{2}\eta^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi \\ S[\Phi] &= \int d^4x\, L(\Phi, \partial_\alpha\Phi) = -\tfrac{1}{2}\int d^4x\, \eta^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi \\ \delta S[\Phi] = 0 \quad &\Rightarrow \quad \Box\Phi \equiv \eta^{\alpha\beta}\partial_\alpha\partial_\beta\Phi = 0 \end{aligned} \tag{1}$$

   (b) Freies massives Feld

$$\begin{aligned} L(\Phi, \partial_\alpha \Phi) &= -\tfrac{1}{2}\eta^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi - \tfrac{1}{2}m^2\Phi^2 \\ S[\Phi] &= \int d^4x\, L(\Phi, \partial_\alpha\Phi) = -\tfrac{1}{2}\int d^4x\, \left(\eta^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi + m^2\Phi^2\right) \\ \delta S[\Phi] = 0 \quad &\Rightarrow \quad \left(\Box - m^2\right)\Phi = 0 \end{aligned} \tag{2}$$

   (c) Feld mit (Selbst-)Wechselwirkung

$$\begin{aligned} L(\Phi, \partial_\alpha \Phi) &= -\tfrac{1}{2}\eta^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi - V(\Phi) \\ S[\Phi] &= \int d^4x\, L(\Phi, \partial_\alpha\Phi) = \int d^4x\, \left(-\tfrac{1}{2}\eta^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi - V(\Phi)\right) \\ \delta S[\Phi] = 0 \quad &\Rightarrow \quad \Box\Phi = V'(\Phi) \equiv \frac{\partial V(\Phi)}{\partial\Phi} \end{aligned} \tag{3}$$

2. Maxwell-Theorie

   (a) Vakuum-Maxwellgleichungen

$$
\begin{aligned}
&L_0(A_\beta, \partial_\alpha A_\beta) = -\tfrac{1}{4} F_{\alpha\beta} F^{\alpha\beta} \\
&S_0[A] = \int d^4x \; L_0(A_\beta, \partial_\alpha A_\beta) = -\tfrac{1}{4} \int d^4x \; F_{\alpha\beta} F^{\alpha\beta} \\
&\delta S_0[A] = 0 \quad \Rightarrow \quad \partial_\alpha F^{\alpha\beta} = 0
\end{aligned}
\tag{4}
$$

   (b) Kopplung an externen Strom J^α (mit $\mu_0 = 1$)

$$
\begin{aligned}
&L(A_\beta, \partial_\alpha A_\beta, J) = L_0(A_\beta, \partial_\alpha A_\beta) + A_\alpha J^\alpha \\
&S[A, J] = \int d^4x \; L(A_\beta, \partial_\alpha A_\beta, J) = S_0[A] + \int d^4x A_\alpha J^\alpha \equiv S_0[A] + S_I[A, J] \\
&\delta S[A, J] = 0 \quad \Rightarrow \quad \partial_\alpha F^{\alpha\beta} = -J^\beta
\end{aligned}
\tag{5}
$$

   (c) Eichinvarianz der Wirkung

$$
\begin{aligned}
&S_0[A_\alpha + \partial_\alpha \Psi] = S_0[A_\alpha] \\
&S_I[A_\alpha + \partial_\alpha \Psi, J] = S_I[A_\alpha, J] \quad \text{(mod Randterme)} \quad \Leftrightarrow \quad \partial_\alpha J^\alpha = 0
\end{aligned}
\tag{6}
$$